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GENERIC THERMAL BUCKLING OF INITIALLY STRESSED ANTISYMMETRIC CROSS-PLY THICK LAMINATES

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Abstract—Equations of motion for antisymmetric cross-ply laminates with thermal effects in a general state of non-uniform initial stress, where the effects of transverse shear and rotary inertia are included, are derived by the Virtual Work theorem. The equations are adjusted to a generic expression by using appropriate transformations. Finally, the thermal stability problems are solved for a simply-supported rectangular laminate in a state of uniform compressive (or tensile) initial stress plus initial bending stress combined with uniform thermal compressive stress plus thermal bending stress. The effects of various parameters on thermal buckling loads are studied.

NOTATION

A_{ij}	extensional stiffnesses
a, b	lengths of laminates in the x- and y-directions, respectively
B_{ij}	coupling stiffnesses
D_{ij}	bending stiffnesses
D^*	generalized rigidity ratio
E_1, E_2	engineering elastic constants
E^*	principal rigidity ratio
G_{12}, G_{23}, G_{13}	engineering shear constants
G^*	transverse shear modulus ratio
N	layer number of laminates
$\bar{N}$	non-dimensional initial stress coefficient
$\bar{N}^T$	thermal buckling coefficient
Q_{ij}	reduced stiffnesses
r	aspect ratio of laminates
S	laminate thickness ratio
t	laminate thickness
u, v, w	displacements of laminates in the x-, y- and z-directions
α_1, α_2	linear coefficients of thermal expansion
α^T	thermal expansion coefficient ratio
β	ratio of initial bending stress to normal stress
β^T	ratio of temperature field
ε	generalized Poisson's ratio
ν_{12}, ν_{21}	Poisson's ratio
ρ	density of laminates
ψ, ϕ	angular changes of lines initially normal to the neutral surface
ω	frequency
Ω	frequency coefficient.

INTRODUCTION

The thermal buckling problems for simply supported thin rectangular plates were solved by using the Raleigh–Ritz energy method by Gossard *et al.*[1]. The Galerkin method has been employed to treat thermal buckling problems for simply supported thin rectangular plates by Van der Neut[2]. Klosner and Forray[3] solved thermal buckling problems of simply supported plates under arbitrarily symmetrical temperature distributions. Many other thermal buckling problems for thin plates can be seen in the books by Boley and Weiner[4] and by Johns[5]. Recently, Prabhu and Durvasula[6, 7] solved thermal buckling problems of thin skew plates for clamped–clamped and restrained boundary conditions by using the Galerkin method. Chen *et al.*[8] employed the Galerkin method to solve the thermal stability problems for simply supported transversely isotropic thick plates in a state of initial stress where the effects of rotary inertia and transverse shear were included.

The equations of motion for a thick plate in a state of initial stress were first made by

Herrmann and Armenakas[9]. Brunelle and Robertson[10], using the Trefftz generalized stress, derived the equations for a thick plate in an arbitrary state of initial stress by using two methods. The equations were used to study the static buckling behaviour of a simply supported thick plate under combined initial compressive stress and initial bending stress acting in the plane of the plate. Research work dealing with the thermal buckling behaviour of thick laminates in an arbitrary state of stress field is not found in the literature.

In this paper, the Virtual Work expression of initially stressed antisymmetric cross-ply thick laminates with thermal effects is derived in a general state of non-uniform initial stress where effects of transverse shear and rotary inertia are included. Because many elastic constants are involved in the thick laminate equations, a transformation[11–18] and some generalized parameters[19, 20] are used in the problems to minimize the number of elastic constants. Thus, the generalized form of equations can be obtained. These equations are then applied to a simply supported rectangular laminate subjected to uniform initial in-plane compression (or tension) and an initial bending stress. The temperature distribution in the laminate is assumed to be uniform plus linear in the transverse direction. The characteristic equations for determining natural frequencies and thermal buckling loads are derived by use of the Galerkin method. The influence of generalized material property parameters and transverse shear modulus and the effect of laminate dimension and the thermal bending stress combined with initial stresses on thermal buckling loads are investigated.

THE VIRTUAL WORK THEOREM

The equations of equilibrium and boundary traction conditions in tensor form can be expressed in terms of Trefftz stress components as[21]

$$[(\delta_{si}+v_{s,j})t^*_{ij}]_{,i}+f^*_s = 0 \tag{1}$$

$$P^*_s = (\delta_{sj}+v_{s,j})t^*_{ij}n_i. \tag{2}$$

If relative extensions and shears are small then the final area and volume are equal to the initial area and volume so that $t^*_{ij} = t_{ij}$, $f^*_s = f_s$, and $P^*_s = P_s$, where t_{ij}, f_s, and P_s are the actual stresses, the body forces and surface tractions, respectively. Following a technique described by Bolotin[22], t_{ij} and v_s are chosen to be the equilibrium large deformation stresses and displacements (i.e. the initial (deformed) state), and $t_{ij}+\sigma_{ij}$ and v_s+u_s are chosen to be the final state values after perturbations σ_{ij} and u_s have taken place. Additionally, f_s and P_s are initial large deformation quantities which become, respectively, $f_s+\Delta f_s+x_s-\rho\ddot{u}_s$ and $P_s+\Delta P_s+p_s$ in the final state. The perturbation stresses σ_{ij} can be decomposed into mechanical and thermal parts so that $\sigma_{ij} = \sigma^M_{ij}+\sigma^T_{ij}$. It is generally assumed that the terms $\sigma^M_{ij}u_{s,j}$ and the initial displacement gradient, $v_{s,j}$ are small enough to drop. Thus, eqns (1) and (2) can be linearized and simplified as

$$[(t_{ij}+\sigma^T_{ij})u_{s,j}+\sigma_{is}]_{,i}+\Delta f_s+x_s-\rho\ddot{u}_s = 0 \tag{3}$$

$$\Delta P_s+p_s = ((t_{ij}+\sigma^T_{ij})u_{s,j}+\sigma_{is})n_i. \tag{4}$$

Oyibo[23] has derived the Virtual Work theorem by multiplying these two equations by the variation of the displacement components δu_s, and then integrating the resulting expression over the initial volume V (eqn (3)) or the surface area σ (eqn (4)). A perturbation strain energy density, $\bar{u}$, defined by $\sigma_{is} = \partial\bar{u}/\partial\varepsilon_{is}$ is introduced. Then, by using the product differentiation rules and the divergence theorem the Virtual Work theorem can be expressed as

$$\delta(\bar{v}+\bar{w})+\int_V \rho\ddot{u}_s\delta u_s\,\mathrm{d}V = \int_{\sigma_p}(\Delta P_s+p_s)\delta u_s\,\mathrm{d}\sigma+\int_V(\Delta f_s+x_s)\delta u_s\,\mathrm{d}V \tag{5}$$

where

$$\delta \bar{v} = \delta \int_V \bar{u}\, \mathrm{d}V = \delta \int_V (\tfrac{1}{2}\sigma_{ij}^M \varepsilon_{ij} + \sigma_{ij}^T \varepsilon_{ij})\, \mathrm{d}V$$

$$\delta \bar{w} = \delta \int_V \tfrac{1}{2}(t_{ij} + \sigma_{ij}^T) u_{s,i} u_{s,j}\, \mathrm{d}V$$

and

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}).$$

This form of Virtual Work theorem will be used subsequently in deriving the needed equations of motion and the associated boundary conditions for antisymmetric cross-ply laminated plates.

GOVERNING EQUATIONS FOR INITIALLY STRESSED ANTISYMMETRIC CROSS-PLY THICK LAMINATES WITH THERMAL EFFECTS

The incremental displacement field is assumed to be of the following form:

$$u_1(x, y, z, \tau) = u(x, y, \tau) + z\psi(x, y, \tau) \tag{6}$$

$$u_2(x, y, z, \tau) = v(x, y, \tau) + z\phi(x, y, \tau) \tag{7}$$

$$u_3(x, y, \tau) = w(x, y, \tau) \tag{8}$$

where u_1 and u_2 are the in-plane displacements and u_3 is the lateral deflection of the neutral surface. u, v, and w are displacements of the neutral surface. ψ and ϕ account for the effect of transverse shear.

The incremental stress-displacement relations are taken to be those of uncoupled linear thermal elasticity. For the plane stress problems only σ_{11}^T and σ_{22}^T exist; all others are zero. The stress-displacement relations for orthotropic plates are

$$\begin{Bmatrix} \sigma_{11}^M \\ \sigma_{22}^M \\ \sigma_{23}^M \\ \sigma_{31}^M \\ \sigma_{12}^M \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \left(\begin{Bmatrix} u_{,x} \\ v_{,y} \\ \phi + w_{,y} \\ \psi + w_{,x} \\ u_{,y} + v_{,x} \end{Bmatrix} + z \begin{Bmatrix} \psi_{,x} \\ \phi_{,y} \\ 0 \\ 0 \\ \psi_{,y} + \phi_{,x} \end{Bmatrix} \right) \tag{9}$$

$$\sigma_{11}^T = -(\alpha_1 Q_{11} + \alpha_2 Q_{12})T, \quad \sigma_{22}^T = -(\alpha_1 Q_{12} + \alpha_2 Q_{22})T \tag{10}$$

where Q_{ij} are the so-called reduced stiffnesses.

The resultant forces and moments acting on antisymmetric cross-ply laminates are obtained by the integration of the stresses in each layer through the laminate thickness

$$(N_x, N_y, Q_y, Q_x, N_{xy}) = \sum_{k=1}^{N} \int_{Z_{k-1}}^{Z_k} (\sigma_{11}^M, \sigma_{22}^M, \sigma_{23}^M, \sigma_{31}^M, \sigma_{12}^M)\, \mathrm{d}z$$

$$(M_x, M_y, Q_y^*, Q_x^*, M_{xy}) = \sum_{k=1}^{N} \int_{Z_{k-1}}^{Z_k} (\sigma_{11}^M, \sigma_{22}^M, \sigma_{23}^M, \sigma_{31}^M, \sigma_{12}^M) z\, \mathrm{d}z$$

$$(N_x^T, N_y^T; M_x^T, M_y^T; M_x^{*T}, M_y^{*T}) = \sum_{k=1}^{N} \int_{Z_{k-1}}^{Z_k} (\sigma_{11}^T, \sigma_{22}^T)(1, z, z^2)\, \mathrm{d}z$$

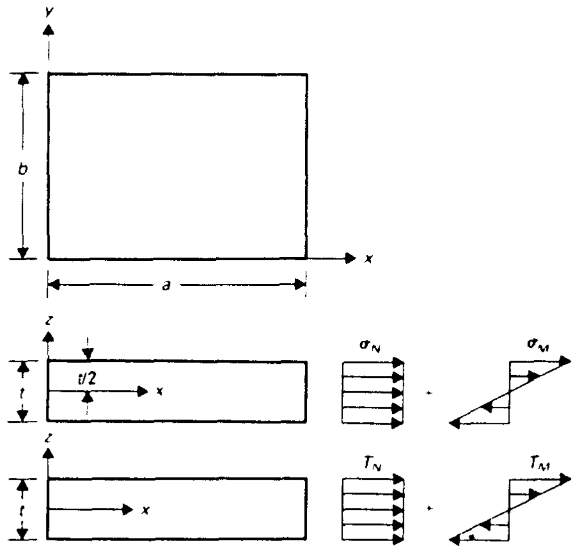

Fig. 1. Rectangular laminated plates subjected to uniform stress plus bending stress in the x-direction with uniform temperature distribution plus linear temperature difference gradient in the z-direction.

$$(\bar{N}_{ij}, \bar{M}_{ij}, \bar{M}^*_{ij}) = \sum_{k=1}^{N} \int_{Z_{k-1}}^{Z_k} t_{ij}(1, z, z^2)\, dz. \tag{11}$$

By expanding the tensor terms of the Virtual Work theorem in eqn (5), using eqns (6)–(11) and the shear correction factor[24], and by utilizing some previous material constant definitions due to Tsai[25] and integrating through the laminate thickness, the Virtual Work theorem can be obtained.

Consider a simply supported antisymmetric cross-ply thick laminate in a state of initial stress. The state of initial stress is

$$t_{11} = \sigma_N + 2z\sigma_M/t \tag{12}$$

with all other initial stresses assumed to be zero. σ_N and σ_M are taken to be constants so that the initial stress field is uniform. It is comprised of a tension (or compression) stress plus bending stress. The plate is subjected to a temperature difference and the temperature difference distribution is

$$T = T_N + 2zT_M/t \tag{13}$$

where T_N and T_M are taken to be constants so that the temperature distribution is a function of the z-coordinate only. This is a possible temperature distribution for plane stress thermal elastic problems[4]. Then, the laminate is subjected to thermal compressive stresses and thermal bending stresses in the x- and y-directions due to the temperature difference in the laminate. This can be seen in Fig. 1. From eqn (11) the only non-zero initial stresses are

$$\begin{aligned} \bar{N}_x &= t\sigma_N \\ \bar{M}_x &= \tfrac{1}{6}t\beta\bar{N}_x \\ \bar{M}^*_x &= \tfrac{1}{12}t^2\bar{N}_x \end{aligned} \tag{14}$$

where $\beta = \sigma_M/\sigma_N$. Also, in this paper, it is assumed that only laminates comprised of laminae of equal thickness are dealt with. From eqn (11), the only non-zero thermal stresses are

$$(N_x^T, N_y^T) = \tfrac{1}{2}N^T(l_1 \pm l_2\beta^T/N)$$

$$(M_x^T, M_y^T) = \tfrac{1}{12}N^T t(l_1\beta^T \pm 3l_2/N)$$

$$(M_x^{*T}, M_y^{*T}) = \tfrac{1}{24}N^T t^2(l_1 \pm 3l_2\beta^T(N^2-2)/N^3) \quad (15)$$

where

$$N^T = t\sigma_N^T, \quad \sigma_N^T = -\alpha_2 Q_{11} T_N, \quad \beta^T = T_M/T_N, \quad l_1 = (\alpha^T \varepsilon D^* \sqrt{E^*} + E^*) + (\alpha^T + \varepsilon D^* \sqrt{E^*}),$$
$$l_2 = (\alpha^T \varepsilon D^* \sqrt{E^*} + E^*) - (\alpha^T + \varepsilon D^* \sqrt{E^*}), \quad \alpha^T = \alpha_1/\alpha_2$$

and

$$D^* = \frac{Q_{12}+2Q_{66}}{\sqrt{(Q_{11}Q_{22})}}, \quad \varepsilon D^* = \frac{Q_{12}}{\sqrt{(Q_{11}Q_{22})}}, \quad E^* = \frac{Q_{22}}{Q_{11}}.$$

By taking variation with respect to the displacement components of the Virtual Work theorem and substituting eqns (12)–(15) into the equation of the Virtual Work theorem, the governing equations and the associated boundary conditions for antisymmetric cross-ply laminated plates subjected to the initial stress and the temperature difference distribution, where the effects of transverse shear and rotary inertia are included, are derived as

$$-(A_{11}+\bar{N}_x+N_x^T)u_{,xx}-(A_{66}+N_y^T)u_{,yy}-(A_{12}+A_{66})v_{,xy}$$
$$-(B_{11}+\bar{M}_x+M_x^T)\psi_{,xx}-M_y^T\psi_{,yy}+\rho t u_{,tt} = 0 \quad (16)$$

$$-(A_{12}+A_{66})u_{,xy}-(A_{66}+\bar{N}_x+N_x^T)v_{,xx}-(A_{11}+N_y^T)v_{,yy}$$
$$-(\bar{M}_x+M_x^T)\phi_{,xx}-(M_y^T-B_{11})\phi_{,yy}+\rho t v_{,tt} = 0 \quad (17)$$

$$-(A_{44}+\bar{N}_x+N_x^T)w_{,xx}-(A_{44}+N_y^T)w_{,yy}-A_{44}\psi_{,x}-A_{44}\phi_{,y}+\rho t w_{,tt} = 0 \quad (18)$$

$$-(B_{11}+\bar{M}_x+M_x^T)u_{,xx}-M_y^T u_{,yy}+A_{44}w_{,x}+A_{44}\psi-(D_{11}+\bar{M}_x^*+M_x^{*T})\psi_{,xx}$$
$$-(D_{66}+M_y^{*T})\psi_{,yy}-(D_{12}+D_{66})\phi_{,xy}+\tfrac{1}{12}\rho t^3\psi_{,tt} = 0 \quad (19)$$

$$-(\bar{M}_x+M_x^T)v_{,xx}-(M_y^T-B_{11})v_{,yy}+A_{44}w_{,y}-(D_{12}+D_{66})\psi_{,xy}$$
$$+A_{44}\phi-(D_{66}+\bar{M}_x^*+M_x^{*T})\phi_{,xx}-(D_{11}+M_y^{*T})\phi_{,yy}+\tfrac{1}{12}\rho t^3\phi_{,tt} = 0 \quad (20)$$

with boundary conditions:

on x = constant edges

$$N_x = (A_{11}+\bar{N}_x+N_x^T)u_{,x}+A_{12}v_{,y}+(B_{11}+\bar{M}_x+M_x^T)\psi_{,x}+N_x^T = 0, \quad v = 0, \quad w = 0$$
$$M_x = (B_{11}+\bar{M}_x+M_x^T)u_{,x}+(D_{11}+\bar{M}_x^*+M_x^{*T})\psi_{,x}+D_{12}\phi_{,y}+M_x^T = 0, \quad \phi = 0; \quad (21)$$

on y = constant edges

$$N_y = A_{12}u_{,x}+(A_{11}+N_y^T)v_{,y}+(M_y^T-B_{11})\phi_{,y}+N_y^T = 0, \quad u = 0, \quad w = 0$$
$$M_y = (M_y^T-B_{11})v_{,y}+D_{12}\psi_{,x}+(D_{11}+M_y^{*T})\phi_{,y}+M_y^T = 0, \quad \psi = 0. \quad (22)$$

SOLUTIONS OF THE EQUATIONS OF MOTION

Assuming the displacement field is in the following form:

$$(u, v, w, \psi, \phi) = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} (u_{mn}, v_{mn}, w_{mn}, \psi_{mn}\bar{m}, \phi_{mn}\bar{n})(C_0S_i, S_iC_0, S_iS_i, C_0S_i, S_iC_0)\, e^{i\omega t} \tag{23}$$

where

$$\bar{m} = \frac{m\pi}{a}, \quad \bar{n} = \frac{n\pi}{b}, \quad C_0 = \cos \bar{m}x, \quad S_i = \sin \bar{n}y.$$

The thermal force N^T in eqns (15) is expressed in double Fourier series form

$$N^T = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} N_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{24}$$

where

$$N_{mn} = \frac{16N^T}{\pi^2 mn} \quad (m, n = 1, 3, \ldots).$$

Since the problem has non-homogeneous boundary conditions due to thermal effects, the displacement field of eqn (23) cannot satisfy both force boundary conditions and moment boundary conditions. The thermal term is expressed as eqn (24), then eqns (23) and (24) satisfy both kinematic and kinetic boundary conditions. The Galerkin method is used to solve this static thermal buckling problem of a simply supported thick laminate under initial stresses. A three term Ritz Galerkin method is employed to obtain the following characteristic equations for the determination of the buckling loads and natural frequencies. Nondimensionalizing the characteristic equations, we can obtain

$$C_{ij}X_i = 0, \quad i, j = 1, 15 \tag{25}$$

where

$$x_1 = u_{11}, \quad x_2 = u_{13}, \quad x_3 = u_{31}, \quad x_4 = v_{11}, \quad x_5 = v_{13}, \quad x_6 = v_{31},$$
$$x_7 = w_{11}, \quad x_8 = w_{13}, \quad x_9 = w_{31}, \quad x_{10} = \psi_{11}, \quad x_{11} = \psi_{13}, \quad x_{12} = \psi_{31},$$
$$x_{13} = \phi_{11}, \quad x_{14} = \phi_{13}, \quad x_{15} = \phi_{31}$$

and

$$C_{ij} = C_{ij}(D^*, E^*, G^*, \varepsilon, r, S, N, \alpha^T, \beta, \beta^T, \bar{N}, \bar{N}^T, \Omega).$$

The coefficients are similar to those of Ref. [18], although they are much more complicated because of thermal effects. The dimensionless parameters used in eqn (25) are

$$A = (1-\varepsilon)D^*\sqrt{E^*}/(1+E^*), \quad A^* = (1+\varepsilon)D^*\sqrt{E^*}/(1+E^*),$$
$$F = (1-(\varepsilon D^*)^2)G^*/(S^2(1+E^*)), \quad F^* = (E^*-1)/(SN(1+E^*)),$$
$$\bar{N} = \frac{\bar{N}_x b^2}{D_{11}\pi^2}, \quad \bar{N}^T = \frac{N^T b^2}{D_{11}\pi^2}, \quad \Omega = \frac{\omega ab}{\pi^2}\sqrt{\left(\frac{\rho t}{D_{11}}\right)}, \quad r = a/b,$$
$$S = t/b, \quad d_1 = l_1 + l_2\beta^T/N, \quad d_2 = r^2(l_1 - l_2\beta^T/N),$$

$$d_3 = l_1\beta^T + 3l_2/N, \quad d_4 = r^2(l_1\beta^T - 3l_2/N),$$
$$d_5 = l_1 + 3l_2\beta^T(N^2-2)/N^3, \quad d_6 = r^2(l_1 - 3l_2\beta^T(N^2-2)/N^3),$$
$$G^* = (G_{23}+G_{31})/E_1.$$

It is easy to derive the two-term and one-term Galerkin solution from eqn (25) by reducing the order of matrix C_{ij} and X_i appropriately.

The basic parameters which exist in eqn (25) include the following group:

(1) parameters of laminate dimension: r, S, N;
(2) parameters of material properties: D^*, ε, E^*, G^*, α^T;
(3) parameters of initial stress: $\bar{N}$, β;
(4) parameters of thermal buckling: $\bar{N}^T$, β^T.

The key parameters of this analysis are generic terms D^*, ε, E^*, and α^T, called the generalized rigidity ratio, generalized Poisson's ratio, principal rigidity ratio and ratio of principal thermal expansion coefficients, respectively. Their limits for almost all orthotropic materials have been established as follows[19, 20]: $0 < D^* \leqslant 1$, $0.12 \leqslant \varepsilon \leqslant 0.65$, $0 < E^* \leqslant 1$, and $0 < \alpha^T \leqslant 1$ (for isotropic materials, $D^* = 1$, $\varepsilon = \nu$, $E^* = 1$, and $\alpha^T = 1$).

RESULTS AND DISCUSSION

The thermal buckling problems for an antisymmetric cross-ply laminate are studied by letting $\Omega = 0$. From the numerous problems solved, only a few typical cases will be selected for discussion. These cases will illustrate the salient features of the ways that the thick laminates considered here behave.

First, the laminates are reduced to transversely isotropic thick plates and thin plates, and the results shown in Fig. 2 are compared with those of Chen *et al.*[8]. In the first case the plates are assumed to be thin by taking $t/a = 0.01$ and $At/b^2\bar{G} = 0.0001$. The large value of $\bar{G}$ implies that there is little transverse shearing. It is expected that the results will correspond to the classical thermal buckling theory of thin plates. In the second case the transversely isotropic thick plates are considered and the thermal bending is taken into account. It can be seen that the present results agree with those solved by Chen *et al.*[8]. These two comparative results are to be expected since eqns (16)–(20) can be reduced easily

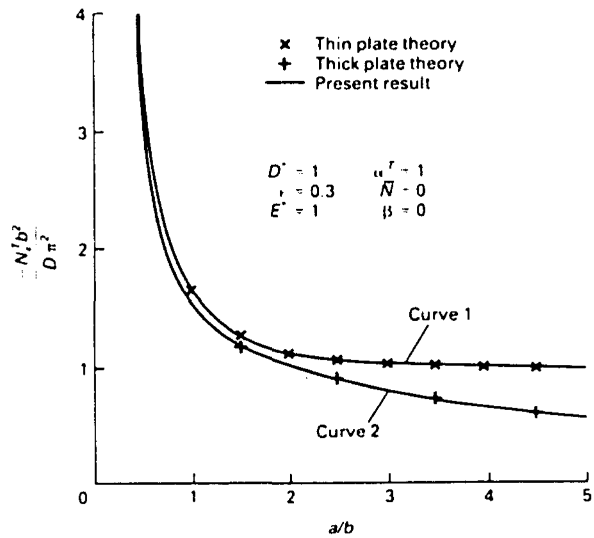

Fig. 2. Comparison of Chen *et al.*'s results with the present results ($\bar{A} = Et/(1-\nu^2)$, $D = Et^3/12(1-\nu^2)$, $\bar{G} = G_{23} = G_{31}$): Curve 1, $t/a = 0.01$, $\bar{A}t/b^2\bar{G} = 0.0001$, $\beta^T = 0$; Curve 2, $t/a = 0.1$, $\bar{A}t/b^2\bar{G} = 0.05$, $\beta^T = 10$.

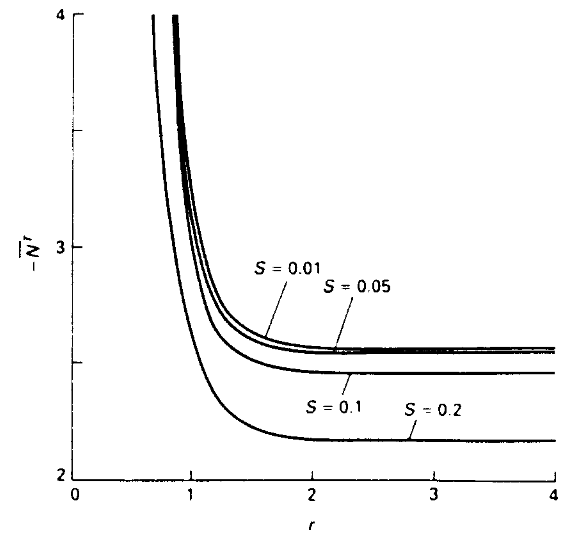

Fig. 3. Thermal buckling coefficient vs r for various S when $D^* = 0.3$, $\varepsilon = 0.3$, $E^* = 0.3$, $G^* = 0.3$, $\alpha^T = 0.3$, $N = 4$, $\bar{N} = 0$, $\beta = 0$, $\beta^T = 0$.

to equations for initially stressed transversely isotropic thick plates with thermal effects and can be further reduced to equations of Classical Plate Theory.

Then, a composite plot of the thermal buckling loads for various thickness ratios is given for antisymmetric cross-ply laminates in Fig. 3. It is shown that the thermal buckling loads decrease with an increase in laminate thickness. The results of $S = 0.01$ compare with the classical plate solutions as used in Fig. 2. The thermal buckling coefficient decreases substantially when S gets larger than 0.05. This means that Classical Lamination Theory is inaccurate for S larger than 0.05. The curves in Fig. 4 reveal the effects of coupling stiffnesses on the laminate. When N increases, the coupling stiffness tends towards zero[26] and the thermal buckling loads of $N = 100$ approach the orthotropic solutions using Classical Lamination Theory. The thermal buckling loads apparently decrease when N gets smaller than 6 where the coupling effects cannot be neglected.

The effects of laminate material properties on thermal buckling coefficients are presented in Figs 5–7. Figures 5 and 6 show that the thermal buckling coefficients decrease with increasing E^* and α^T, respectively, along a fixed D^* curve and the effects of D^* on thermal buckling coefficients are not so apparent as those of E^* and α^T. However, due to

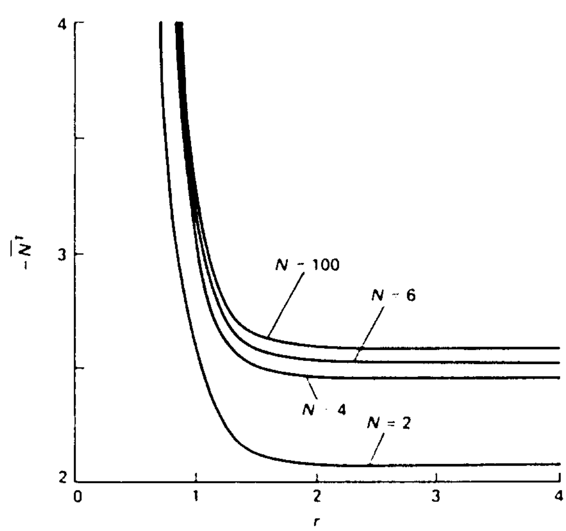

Fig. 4. Thermal buckling coefficient vs r for various N when $D^* = 0.3$, $\varepsilon = 0.3$, $E^* = 0.3$, $G^* = 0.3$, $\alpha^T = 0.3$, $S = 0.1$, $\bar{N} = 0$, $\beta = 0$, $\beta^T = 0$.

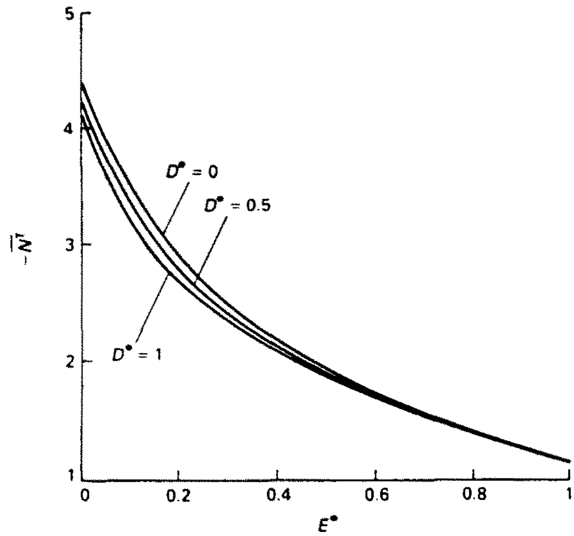

Fig. 5. Thermal buckling coefficient vs E^* for various D^* when $\varepsilon = 0.3$, $G^* = 0.3$, $\alpha^T = 0.3$, $N = 4$, $S = 0.1$, $\bar{N} = 0$, $\beta = 0$, $\beta^T = 0$, $r = 2$.

the bounded limits of D^* values, the thermal buckling coefficients of composite materials with other D^* values can only be estimated from these charts. Figure 7 depicts the thermal buckling coefficients reduced almost linearly with increasing ε for a fixed G^*. It can also be seen that the thermal buckling coefficients increase with an increase in transverse shear modulus G^*, especially, for small values of G^* ($G^* < 1$). A large value of G^* implies that there is little transverse shearing. Therefore, as G^* gets larger, the results will compare with the classical thermal buckling solutions.

Plots are made of $-\bar{N}^T$ vs initial stress $\bar{N}$ in Fig. 8. It is shown that the thermal buckling loads increase almost linearly with increasing $\bar{N}$ for a fixed value of D^*. It is because the compressive stress will reduce the strength of the laminate. As $\bar{N}^T = 0$, i.e. the thermal effect is excluded, the laminate will buckle due to initial compressive stress. In Fig. 9, plots are made of $-\bar{N}^T$ vs β^T for various $\bar{N}$ and β. It is shown that the thermal buckling strength will be reduced when the thermal moment is increased. It can be seen that the effects of initial moment on thermal buckling loads are not as significant as those of thermal moment.

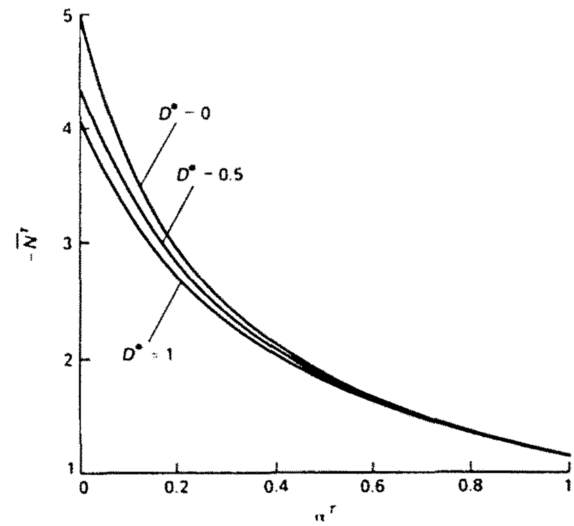

Fig. 6. Thermal buckling coefficient vs α^T for various D^* when $\varepsilon = 0.3$, $E^* = 0.3$, $G^* = 0.3$, $r = 2$, $N = 4$, $S = 0.1$, $\bar{N} = 0$, $\beta = 0$, $\beta^T = 0$.

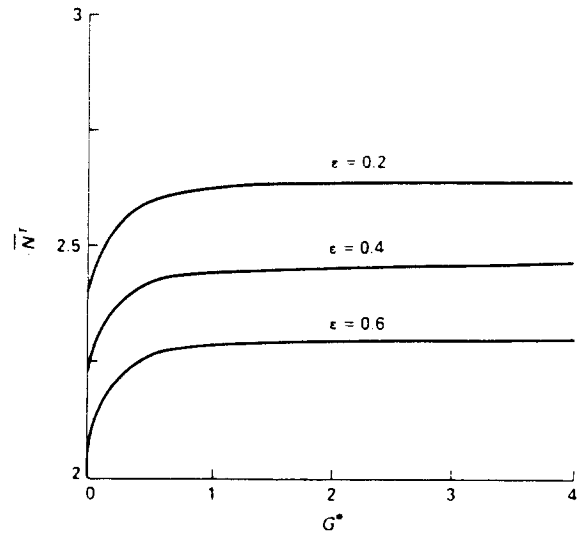

Fig. 7. Thermal buckling coefficient vs G^* for various ε when $D^* = 0.3$, $E^* = 0.3$, $\alpha^r = 0.3$, $r = 2$, $N = 4$, $S = 0.1$, $\bar{N} = 0$, $\beta = 0$, $\beta^r = 0$.

CONCLUSION

The governing equations of the thermal elastic antisymmetric cross-ply thick laminate have been derived. The transverse shear deformations and rotary inertia are considered for this thick laminate formulation. These can be reduced subsequently for a thermal elastic transversely isotropic thick plate and for a thin plate. The numerical results show that the present Thick Lamination Theory reasonably corresponds to transversely isotropic Thick Plate and Classical Thin Plate Theory.

The preceding results reveal the points given below.

(1) The thicker the laminate is the lower the thermal buckling load.

(2) The thermal buckling load increases with an increase of the number of layers.

(3) The thermal buckling load decreases greatly with increasing E^* or α^r.

(4) The D^* does not have significant effects on thermal stability but it provides bounded values of thermal buckling coefficients for all orthotropic materials.

(5) The larger the generalized Poisson's ratio ε, the lower the thermal buckling load.

(6) The transverse shear effects can decrease the thermal stability.

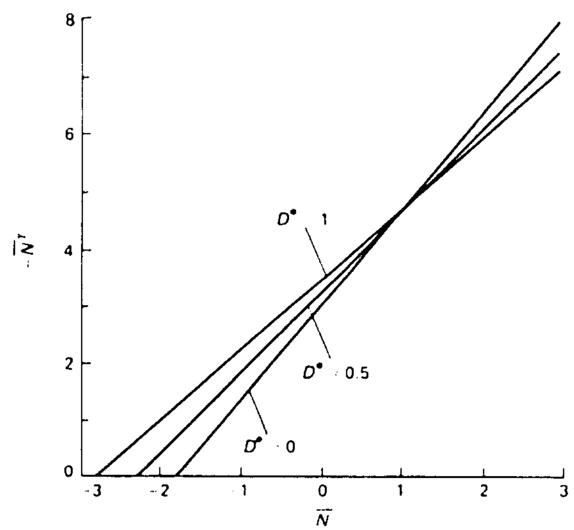

Fig. 8. Thermal buckling coefficient vs $\bar{N}$ for various D^* when $\varepsilon = 0.3$, $E^* = 0.3$, $G^* = 0.3$, $\alpha^r = 0.3$, $r = 1$, $S = 0.1$, $\bar{N} = 0$, $\beta = 0$, $\beta^r = 0$.

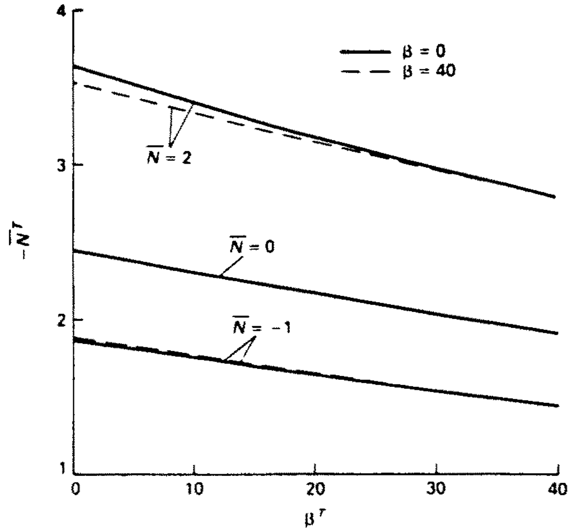

Fig. 9. Thermal buckling coefficient vs β^T for various $\bar{N}$ and β when $D^* = 0.3$, $\varepsilon = 0.3$, $E^* = 0.3$, $G^* = 0.3$, $\alpha^T = 0.3$, $r = 2$, $N = 4$, $S = 0.1$.

(7) The thermal buckling loads increase almost linearly with an increase of initial stress from compression to tension.

(8) The thermal bending moments significantly reduce the thermal buckling load of the thick laminate. However, the initial bending stresses have little effects on thermal stability.

The results presented are limited by the features of antisymmetric cross-ply laminates. In addition, the laminate is initially assumed to be flat. From the procedure used to fabricate most current-day laminates, a six-layer antisymmetric cross-ply laminate would actually be cylindrical[27]. However, the results indicate some of the many interesting effects that can be studied with this analysis. There is a lot of room for future study of the effects that other thermal stress fields and other laminated plates would have on this and other geometries.

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